EQUATIONS

Principal stresses for plane stress state

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Extreme-value shear stresses for plane stress state

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Plane-stress transformation equations

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi$$

Axial stress

$$\sigma = \frac{F}{A}$$

Normal stress for beam in bending

$$\sigma = \frac{Mc}{I}$$

Second-area moment

- for rectangular cross-section

$$I = \frac{bh^3}{12}$$

- for circular cross-section

$$I = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

- for hollow round cross-section

$$I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

Maximum transverse shear stress

- for rectangular cross-section

$$\tau_{\max} = \frac{3V}{2A}$$

- for circular cross-section

$$\tau_{\max} = \frac{4V}{3A}$$

- for hollow, thin-walled round cross-section

$$\tau_{\max} = \frac{2V}{A}$$

- for thin-walled I-beam

$$\tau_{\max} \approx \frac{V}{A_{\text{web}}}$$

EQUATIONS (continued)

Shear stress due to torsion

$$\tau = \frac{T\rho}{J}$$

$$\tau_{\max} = \frac{Tr}{J}$$

Polar second moment of area

- for circular cross-section

$$J = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

- for hollow round cross-section

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

Ductile Coulomb-Mohr (DCM) theory

$$\frac{1}{n} = \frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}$$

Maximum shear stress (MSS) theory for ductile materials

$$n = \frac{S_y}{2\tau_{\max}} = \frac{S_y}{\sigma_1 - \sigma_3}$$

Distortion energy (DE) theory for ductile materials

$$n = \frac{S_y}{\sigma'}$$

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}$$

$$\sigma' = \sqrt{\frac{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_x - \sigma_z)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)}{2}}$$

Brittle Coulomb-Mohr (BCM) theory

$$\frac{1}{n} = \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}}$$

Modified Mohr (MM) theory for brittle materials (plane stress)

$$n = \frac{S_{ut}}{\sigma_A}$$ for $\sigma_A \geq \sigma_B \geq 0$ and for $\sigma_A \geq 0 \geq \sigma_B$ where $\left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$

$$\frac{1}{n} = \frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}}$$ for $\sigma_A \geq 0 \geq \sigma_B$ where $\left|\frac{\sigma_B}{\sigma_A}\right| > 1$

$$n = -\frac{S_{uc}}{\sigma_B}$$ for $0 \geq \sigma_A \geq \sigma_B$

Stress intensity factor

$$K_I = \beta\sigma\sqrt{\pi a}$$

Factor of safety against sudden fracture

$$n = \frac{K_{Ic}}{K_I}$$

Road maps and important design equations for the Stress-Life Method

As stated in Section 6–16, there are three categories of fatigue problems. The important procedures and equations for deterministic stress-life problems are presented here, organized into those three categories.

Completely Reversing Simple Loading

1 Determine S_e' either from test data or

$$S_e' = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \tag{6–10}$$

2 Modify S_e' to determine S_e.

$$S_e = k_a k_b k_c k_d k_e S_e' \tag{6–17}$$

$$k_a = aS_{ut}^b \tag{6–18}$$

Table 6–2 Curve Fit Parameters for Surface Factor, Equation (6–18)

Surface Finish	Factor *a*		Exponent *b*
	S_{ut}, kpsi	S_{ut}, MPa	
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

Rotating shaft. For bending or torsion,

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < 254 \text{ mm} \end{cases} \tag{6–19}$$

For axial,

$$k_b = 1 \tag{6–20}$$

Nonrotating member. For bending, use Table 6–3 for d_e and substitute into Equation (6–19) for d.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}25)$$

$$S_T/S_{RT} = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2$$
$$S_T/S_{RT} = 0.99 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2 \qquad (6\text{–}26)$$

Either use the ultimate strength from Equation (6–26) to estimate S_e at the operating temperature, with $k_d = 1$, or use the known S_e at room temperature with $k_d = S_T/S_{RT}$ from Equation (6–26).

Table 6–4 Reliability Factor k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702

3 Determine fatigue stress-concentration factor, K_f or K_{fs}. First, find K_t or K_{ts} from Table A–15.

$$K_f = 1 + q(K_t - 1) \qquad \text{or} \qquad K_{fs} = 1 + q_s(K_{ts} - 1) \qquad (6\text{–}32)$$

Obtain q from either Figure 6–26 or 6–27.

Alternatively,

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \qquad (6\text{–}34)$$

Bending or axial:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad 50 \le S_{ut} \le 250 \text{ kpsi}$$
$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \qquad 340 \le S_{ut} \le 1700 \text{ MPa} \qquad (6\text{–}35)$$

Torsion:

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad 50 \le S_{ut} \le 220 \text{ kpsi}$$
$$\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \qquad 340 \le S_{ut} \le 1500 \text{ MPa} \qquad (6\text{–}36)$$

4 Apply K_f to the nominal completely reversed stress, $\sigma_a = K_f \sigma_{a0}$.

5 Determine f from Figure 6–23 or Equation (6–11). For S_{ut} lower than the range, use $f = 0.9$.

$$f = 1.06 - 2.8(10^{-3})S_{ut} + 6.9(10^{-6})S_{ut}^2 \qquad 70 < S_{ut} < 200 \text{ kpsi}$$
$$f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \qquad 500 < S_{ut} < 1400 \text{ MPa} \tag{6–11}$$

$$a = (f\,S_{ut})^2/S_e \tag{6–13}$$

$$b = -[\log\,(f\,S_{ut}/S_e)]/3 \tag{6–14}$$

6 Determine fatigue strength S_f at N cycles, or, N cycles to failure at a reversing stress σ_{ar}.

(*Note:* This only applies to purely reversing stresses where $\sigma_m = 0$.)

$$S_f = aN^b \tag{6–12}$$

$$N = (\sigma_{ar}/a)^{1/b} \tag{6–15}$$

Fluctuating Simple Loading

For S_e, K_f or K_{fs}, see previous subsection.

1 Calculate σ_m and σ_a. Apply K_f to both stresses.

$$\sigma_a = |\sigma_{max} - \sigma_{min}|/2 \qquad \sigma_m = (\sigma_{max} + \sigma_{min})/2 \tag{6–8), (6–9}$$

2 Check for infinite life with a fatigue failure criterion. Use Goodman criterion for conservative result, or another criterion from Section 6–13.

$$\sigma_m \geq 0 \qquad n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} \tag{6–41}$$

$$\sigma_m < 0 \qquad n_f = \frac{S_e}{\sigma_a} \tag{6–42}$$

3 Check for localized yielding.

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{S_y}{\sigma_a + |\sigma_m|} \tag{6–43}$$

4 For finite-life, find an equivalent completely reversed stress to use on the *S-N* diagram with Equation (6–15). Select one of the following criterion. Discussion of merits is in Section 6–14.

Goodman: $$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/S_{ut}} \quad (6\text{–}59)$$

Morrow: $$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\tilde{\sigma}_f} \quad \text{or} \quad \sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\sigma'_f} \quad (6\text{–}60)$$

Estimate for steel: $$\sigma'_f = S_{ut} + 50 \text{ kpsi} \quad \text{or} \quad \sigma'_f = S_{ut} + 345 \text{ MPa} \quad (6\text{–}44)$$

SWT: $$\sigma_{ar} = \sqrt{\sigma_{\max}\sigma_a} = \sqrt{(\sigma_m + \sigma_a)\sigma_a} \quad (6\text{–}61)$$

Walker: $$\sigma_{ar} = \sigma_{\max}^{1-\gamma}\sigma_a^{\gamma} = (\sigma_m + \sigma_a)^{1-\gamma}\sigma_a^{\gamma} \quad (6\text{–}62)$$

Estimate for steel:
$$\gamma = -0.0002S_{ut} + 0.8818 \quad (S_{ut} \text{ in MPa})$$
$$\gamma = -0.0014S_{ut} + 0.8818 \quad (S_{ut} \text{ in kpsi}) \quad (6\text{–}57)$$

If determining the finite life N with a factor of safety n, substitute σ_{ar}/n for σ_{ar} in Equation (6–15). That is,

$$N = \left(\frac{\sigma_{ar}/n}{a}\right)^{1/b}$$

Combination of Loading Modes

See previous subsections for earlier definitions.

1 Calculate von Mises stresses for alternating and mean stress states, σ'_a and σ'_m. When determining S_e, do not use K_c nor divide by K_f or K_{fs}. Apply K_f and/or K_{fs} directly to each specific alternating and mean stress. For the special case of combined bending, torsional shear, and axial stresses

$$\sigma'_a = \{[(K_f)_{\text{bending}}(\sigma_{a0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{a0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{a0})_{\text{torsion}}]^2\}^{1/2} \quad (6\text{–}66)$$

$$\sigma'_m = \{[(K_f)_{\text{bending}}(\sigma_{m0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{m0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{m0})_{\text{torsion}}]^2\}^{1/2} \quad (6\text{–}67)$$

2 Apply stresses to fatigue criterion [see previous subsection].

3 To check for yielding, apply the distortion energy theory as usual by putting maximum stresses on a stress element and calculating a von Mises stress.

$$n_y = S_y / \sigma'_{\max}$$

Or for a conservative check, apply the yield line with the von Mises stresses for alternating and mean stresses.

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} \quad (6\text{–}43)$$

EQUATIONS (continued)

Key design (shear failure due to static loading)

$$n = \frac{S_{sy}}{\tau} = \frac{0.577 S_y}{\tau}$$

$$\tau = \frac{F}{A_{\text{shear}}} = \frac{T/r}{w * l}$$

Key design (crushing failure due to static loading)

$$n = \frac{S_{yc}}{\sigma}$$

$$\sigma = \frac{T/r}{\frac{h}{2} * l}$$

EQUATIONS (continued)

Helical compression springs

$$k = \frac{F}{y} \approx \frac{d^4 G}{8D^3 N_a}$$
$$C = \frac{D}{d}$$
$$\tau = K_B \frac{8FD}{\pi d^3}$$
$$K_B = \frac{4C + 2}{4C - 3}$$
$$S_{ut} = \frac{A}{d^m}$$
$$L_{\text{crit}} = \frac{\pi D}{\alpha} \sqrt{\frac{2(E - G)}{2G + E}}$$
$$L_{\text{crit}} = 2.63 \frac{D}{\alpha} \text{ for steels}$$
$$n_s = \frac{S_{sy}}{\tau}$$
$$\frac{1}{n_f} = \frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}$$
$$\tau_a = K_B \frac{8F_a D}{\pi d^3}$$
$$\tau_m = K_B \frac{8F_m D}{\pi d^3}$$
$$F_a = \frac{F_{\max} - F_{\min}}{2}$$
$$F_m = \frac{F_{\max} + F_{\min}}{2}$$
$$S_{su} = 0.67 S_{ut}$$
$$S_{se} = \frac{S_{sa}}{1 - \frac{S_{sm}}{S_{su}}}$$

For unpeened springs: $S_{sa} = 35$ ksi = 241 MPa and $S_{sm} = 55$ ksi = 379 MPa

For peened springs: $S_{sa} = 57.5$ ksi = 398 MPa and $S_{sm} = 77.5$ ksi = 534 MPa